1

Physics and Measurement

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ1.1 The meterstick measurement, (a), and (b) can all be 4.31 cm. The meterstick measurement and (c) can both be 4.24 cm. Only (d) does not overlap. Thus (a), (b), and (c) all agree with the meterstick measurement.

OQ1.2 Answer (d). Using the relation

$$1 \text{ ft} = 12 \text{ in}\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right) = 0.304\ 8 \text{ m}$$

we find that

$$1\ 420 \text{ ft}^2\left(\frac{0.304\ 8 \text{ m}}{1 \text{ ft}}\right)^2 = 132 \text{ m}^2$$

OQ1.3 The answer is yes for (a), (c), and (e). You cannot add or subtract a number of apples and a number of jokes. The answer is no for (b) and (d). Consider the gauge of a sausage, 4 kg/2 m, or the volume of a cube, $(2 \text{ m})^3$. Thus we have (a) yes; (b) no; (c) yes; (d) no; and (e) yes.

OQ1.4 41 € ≈ 41 € (1 L/1.3 €)(1 qt/1 L)(1 gal/4 qt) ≈ (10/1.3) gal ≈ 8 gallons, answer (c).

OQ1.6 The number of decimal places in a sum of numbers should be the same as the smallest number of decimal places in the numbers summed.

$$\begin{array}{r} 21.4\ \text{s} \\ 15\ \text{s} \\ 17.17\ \text{s} \\ 4.003\ \text{s} \\ \hline 57.573\ \text{s} \end{array} = 58\ \text{s, answer (d).}$$

OQ1.7 The population is about 6 billion = 6×10^9. Assuming about 100 lb per person = about 50 kg per person (1 kg has the weight of about 2.2 lb), the total mass is about $(6 \times 10^9)(50\ \text{kg}) = 3 \times 10^{11}$ kg, answer (d).

OQ1.8 No: A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct.

Yes: If an equation is not dimensionally correct, it cannot be correct.

OQ1.9 Mass is measured in kg; acceleration is measured in m/s^2. Force = mass × acceleration, so the units of force are answer (a) $\text{kg}\cdot\text{m/s}^2$.

OQ1.10 0.02(1.365) = 0.03. The result is $(1.37 \pm 0.03) \times 10^7$ kg. So (d) 3 digits are significant.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ1.1 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

CQ1.2 The metric system is considered superior because units larger and smaller than the basic units are simply related by multiples of 10. Examples: 1 km = 10^3 m, 1 mg = 10^{-3} g = 10^{-6} kg, 1 ns = 10^{-9} s.

CQ1.3 A unit of time should be based on a reproducible standard so it can be used everywhere. The more accuracy required of the standard, the less the standard should change with time. The current, very accurate standard is the period of vibration of light emitted by a cesium atom. Depending on the accuracy required, other standards could be: the period of light emitted by a different atom, the period of the swing of a pendulum at a certain place on Earth, the period of vibration of a sound wave produced by a string of a specific length, density, and tension, and the time interval from full Moon to full Moon.

CQ1.4 (a) 0.3 millimeters; (b) 50 microseconds; (c) 7.2 kilograms

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 1.1 Standards of Length, Mass, and Time

P1.1 (a) Modeling the Earth as a sphere, we find its volume as

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37\times10^6\ \text{m}\right)^3 = 1.08\times10^{21}\ \text{m}^3$$

Its density is then

$$\rho = \frac{m}{V} = \frac{5.98\times10^{24}\ \text{kg}}{1.08\times10^{21}\ \text{m}^3} = \boxed{5.52\times10^3\ \text{kg/m}^3}$$

(b) This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2000 to 3000 kg/m^3. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.2 With V = (base area)(height), $V = \left(\pi r^2\right)h$ and $\rho = \dfrac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1\ \text{kg}}{\pi(19.5\ \text{mm})^2(39.0\ \text{mm})}\left(\frac{10^9\ \text{mm}^3}{1\ \text{m}^3}\right)$$

$$\rho = \boxed{2.15\times10^4\ \text{kg/m}^3}$$

P1.3 Let V represent the volume of the model, the same in $\rho = \dfrac{m}{V}$, for both.

Then $\rho_{\text{iron}} = 9.35\ \text{kg}/V$ and $\rho_{\text{gold}} = \dfrac{m_{\text{gold}}}{V}$.

Next, $$\frac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \frac{m_{\text{gold}}}{9.35\ \text{kg}}$$

and $$m_{\text{gold}} = (9.35\ \text{kg})\left(\frac{19.3\times10^3\ \text{kg/m}^3}{7.87\times10^3\ \text{kg/m}^3}\right) = \boxed{22.9\ \text{kg}}$$

P1.4 (a) $\rho = m/V$ and $V = (4/3)\pi r^3 = (4/3)\pi(d/2)^3 = \pi d^3/6$, where d is the diameter.

Then $$\rho = 6m/\pi d^3 = \frac{6\left(1.67\times10^{-27}\text{kg}\right)}{\pi\left(2.4\times10^{-15}\text{m}\right)^3} = \boxed{2.3\times10^{17}\text{kg/m}^3}$$

(b) $$\frac{2.3\times10^{17}\ \text{kg/m}^3}{22.6\times10^3\ \text{kg/m}^3} = \boxed{1.0\times10^{13}\ \text{times the density of osmium}}$$

P1.5 For either sphere the volume is $V = \frac{4}{3}\pi r^3$ and the mass is $m = \rho V = \rho \frac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s} = \frac{\rho(4/3)\pi r_\ell^3}{\rho(4/3)\pi r_s^3} = \frac{r_\ell^3}{r_s^3} = 5$$

Then $r_\ell = r_s\sqrt[3]{5} = (4.50\text{ cm})\sqrt[3]{5} = \boxed{7.69\text{ cm}}$

***P1.6** The volume of a spherical shell can be calculated from

$$V = V_o - V_i = \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$$

From the definition of density, $\rho = \frac{m}{V}$, so

$$m = \rho V = \rho\left(\frac{4}{3}\pi\right)\left(r_2^3 - r_1^3\right) = \boxed{\frac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}}$$

Section 1.2 Matter and Model Building

P1.7 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance L = 0.200 nm, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141\text{ nm}}$.

P1.8 (a) Treat this as a conversion of units using 1 Cu-atom = 1.06×10^{-25} kg, and 1 cm = 10^{-2} m:

$$\text{density} = \left(8\ 920\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right)^3\left(\frac{\text{Cu-atom}}{1.06\times10^{-25}\text{ kg}}\right)$$

$$= \boxed{8.42\times10^{22}\ \frac{\text{Cu-atom}}{\text{cm}^3}}$$

(b) Thinking in terms of units, invert answer (a):

$$(\text{density})^{-1} = \left(\frac{1\ \text{cm}^3}{8.42 \times 10^{22}\ \text{Cu-atoms}}\right)$$

$$= \boxed{1.19 \times 10^{-23}\ \text{cm}^3/\text{Cu-atom}}$$

(c) For a cube of side L,

$$L^3 = 1.19 \times 10^{-23}\ \text{cm}^3 \rightarrow L = \boxed{2.28 \times 10^{-8}\text{cm}}$$

Section 1.3 Dimensional Analysis

P1.9 (a) Write out dimensions for each quantity in the equation

$$v_f = v_i + ax$$

The variables v_f and v_i are expressed in units of m/s, so

$$[v_f] = [v_i] = \text{LT}^{-1}$$

The variable a is expressed in units of m/s^2; $[a] = \text{LT}^{-2}$

The variable x is expressed in meters. Therefore, $[ax] = \text{L}^2\text{T}^{-2}$

Consider the right-hand member (RHM) of equation (a):

$$[\text{RHM}] = \text{LT}^{-1} + \text{L}^2\text{T}^{-2}$$

Quantities to be added must have the same dimensions.
Therefore, equation (a) **is not** dimensionally correct.

(b) Write out dimensions for each quantity in the equation

$$y = (2\ \text{m}) \cos (kx)$$

For y, $[y] = \text{L}$

for 2 m, $[2\ \text{m}] = \text{L}$

and for (kx), $[kx] = \left[\left(2\ \text{m}^{-1}\right)x\right] = \text{L}^{-1}\text{L}$

Therefore we can think of the quantity kx as an angle in radians, and we can take its cosine. The cosine itself will be a pure number with no dimensions. For the left-hand member (LHM) and the right-hand member (RHM) of the equation we have

$$[\text{LHM}] = [y] = \text{L} \qquad [\text{RHM}] = [2\,\text{m}][\cos (kx)] = \text{L}$$

These are the same, so equation (b) **is** dimensionally correct.

P1.10 Circumference has dimensions L, area has dimensions L^2, and volume has dimensions L^3. Expression (a) has dimensions $L(L^2)^{1/2} = L^2$, expression (b) has dimensions L, and expression (c) has dimensions $L(L^2) = L^3$. $\boxed{\text{The matches are: (a) and (f), (b) and (d), and (c) and (e).}}$

P1.11 (a) Consider dimensions in terms of their mks units. For kinetic energy *K*:

$$[K] = \left[\left(\frac{p^2}{2m}\right)\right] = \frac{[p]^2}{kg} = \frac{kg\cdot m^2}{s^2}$$

Solving for $[p^2]$ and $[p]$ then gives

$$[p]^2 = \frac{kg^2\cdot m^2}{s^2} \quad \to \quad [p] = \frac{kg\cdot m}{s}$$

$\boxed{\text{The units of momentum are kg}\cdot\text{m/s.}}$

(b) Momentum is to be expressed as the product of force (in N) and some other quantity *X*. Considering dimensions in terms of their mks units,

$$[\mathrm{N}]\cdot[X] = [p]$$
$$\frac{\mathrm{kg\cdot m}}{\mathrm{s}^2}\cdot[X] = \frac{\mathrm{kg\cdot m}}{\mathrm{s}}$$
$$[X] = \mathrm{s}$$

Therefore, the units of momentum are $\boxed{\mathrm{N\cdot s}}$.

P1.12 We substitute $[\mathrm{kg}] = [M]$, $[\mathrm{m}] = [L]$, and $[F] = \left[\frac{\mathrm{kg\cdot m}}{\mathrm{s}^2}\right] = \frac{[M][L]}{[T]^2}$ into Newton's law of universal gravitation to obtain

$$\frac{[M][L]}{[T]^2} = \frac{[G][M]^2}{[L]^2}$$

Solving for $[G]$ then gives

$$[G] = \frac{[L]^3}{[M][T]^2} = \boxed{\frac{\mathrm{m}^3}{\mathrm{kg\cdot s}^2}}$$

***P1.13** The term *x* has dimensions of L, *a* has dimensions of LT^{-2}, and *t* has dimensions of T. Therefore, the equation $x = ka^m t^n$ has dimensions of

$$\mathrm{L} = \left(\mathrm{LT}^{-2}\right)^m (\mathrm{T})^n \quad \text{or} \quad \mathrm{L}^1\mathrm{T}^0 = \mathrm{L}^m\mathrm{T}^{n-2m}$$

The powers of L and T must be the same on each side of the equation.

Therefore,

$$\mathrm{L}^1 = \mathrm{L}^m \text{ and } \boxed{m = 1}$$

Likewise, equating terms in T, we see that $n - 2m$ must equal 0. Thus, $\boxed{n = 2}$. The value of k, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

P1.14 Summed terms must have the same dimensions.

(a) $[\mathrm{X}] = [At^3] + [Bt]$

$$\mathrm{L} = [A]\mathrm{T}^3 + [B]\mathrm{T} \rightarrow \boxed{[A] = \mathrm{L}/\mathrm{T}^3, \text{ and } [B] = \mathrm{L}/\mathrm{T}}.$$

(b) $[dx/dt] = \left[3At^2\right] + [B] = \boxed{\mathrm{L}/\mathrm{T}}$.

Section 1.4 Conversion of Units

P1.15 From Table 14.1, the density of lead is 1.13×10^4 kg/m^3, so we should expect our calculated value to be close to this value. The density of water is 1.00×10^3 kg/m^3, so we see that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as $\rho = m/V$. We must convert to SI units in the calculation.

$$\begin{aligned}\rho &= \left(\frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\right)\left(\frac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 \\ &= \left(\frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\right)\left(\frac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\frac{1\,000\,000 \text{ cm}^3}{1 \text{ m}^3}\right) \\ &= \boxed{1.14 \times 10^4 \text{ kg/m}^3}\end{aligned}$$

Observe how we set up the unit conversion fractions to divide out the units of grams and cubic centimeters, and to make the answer come out in kilograms per cubic meter. At one step in the calculation, we note that **one million** cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference from the tabulated values is possibly due to measurement uncertainty and does not indicate a discrepancy.

P1.16 The weight flow rate is

$$\left(1\ 200\frac{\text{ton}}{\text{h}}\right)\left(\frac{2000\text{ lb}}{\text{ton}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=\boxed{667\text{ lb/s}}$$

P1.17 For a rectangle, Area = Length × Width. We use the conversion 1 m = 3.281 ft. The area of the lot is then

$$A=LW=(75.0\text{ ft})\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)(125\text{ ft})\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)=\boxed{871\text{ m}^2}$$

P1.18 Apply the following conversion factors: 1 in = 2.54 cm, 1 d = 86 400 s, 100 cm = 1m, and 10^9 nm = 1 m. Then, the rate of hair growth per second is

$$\text{rate}=\left(\frac{1}{32}\text{ in/day}\right)\frac{(2.54\text{ cm/in})\left(10^{-2}\text{ m/cm}\right)\left(10^{9}\text{ nm/m}\right)}{86\ 400\text{ s/day}}$$
$$=\boxed{9.19\text{ nm/s}}$$

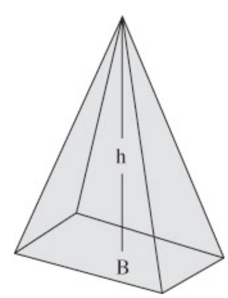
This means the proteins are assembled at a rate of many layers of atoms each second!

P1.19 The area of the four walls is (3.6 + 3.8 + 3.6 + 3.8) m × (2.5 m) = 37 m^2. Each sheet in the book has area (0.21 m)(0.28 m) = 0.059 m^2. The number of sheets required for wallpaper is 37 m^2/0.059 m^2 = 629 sheets = 629 sheets(2 pages/1 sheet) = 1260 pages.

The number of pages in Volume 1 are insufficient.

P1.20 We use the formula for the volume of a pyramid given in the problem and the conversion 43 560 ft^2 = 1 acre. Then,

$$V=Bh$$
$$=\frac{1}{3}\left[(13.0\text{ acres})\left(43\ 560\text{ ft}^2\text{/acre}\right)\right]\times(481\text{ ft})$$
$$=9.08\times10^{7}\text{ ft}^3$$



ANS FIG. P1.20

or

$$V=\left(9.08\times10^{7}\text{ ft}^3\right)\left(\frac{2.83\times10^{-2}\text{ m}^3}{1\text{ ft}^3}\right)$$
$$=\boxed{2.57\times10^{6}\text{ m}^3}$$

P1.21 To find the weight of the pyramid, we use the conversion 1 ton = 2 000 lbs:

$$F_g = (2.50 \text{ tons/block})(2.00\times10^6 \text{ blocks})(2\,000 \text{ lb/ton})$$
$$= \boxed{1.00\times10^{10} \text{ lbs}}$$

P1.22 (a) $$\text{rate} = \left(\frac{30.0 \text{ gal}}{7.00 \text{ min}}\right)\left(\frac{1 \text{ mi}}{60 \text{ s}}\right) = \boxed{7.14\times10^{-2}\,\frac{\text{gal}}{\text{s}}}$$

(b) $$\text{rate} = 7.14\times10^{-2}\,\frac{\text{gal}}{\text{s}}\left(\frac{231 \text{ in}^3}{1 \text{ gal}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3$$
$$= \boxed{2.70\times10^{-4}\,\frac{\text{m}^3}{\text{s}}}$$

(c) To find the time to fill a 1.00-m^3 tank, find the rate time/volume:

$$2.70\times10^{-4}\,\frac{\text{m}^3}{\text{s}} = \left(\frac{2.70\times10^{-4} \text{ m}^3}{1 \text{ s}}\right)$$

or $$\left(\frac{2.70\times10^{-4} \text{ m}^3}{1 \text{ s}}\right)^{-1} = \left(\frac{1 \text{ s}}{2.70\times10^{-4} \text{ m}^3}\right) = 3.70\times10^3\,\frac{\text{s}}{\text{m}^3}$$

and so: $$3.70\times10^3 \text{ s}\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = \boxed{1.03 \text{ h}}$$

***P1.23** It is often useful to remember that the 1 600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1 609 meters in a mile. Thus, 1 acre is equal in area to

$$(1 \text{ acre})\left(\frac{1 \text{ mi}^2}{640 \text{ acres}}\right)\left(\frac{1\,609 \text{ m}}{\text{mi}}\right)^2 = \boxed{4.05\times10^3 \text{ m}^2}$$

***P1.24** The volume of the interior of the house is the product of its length, width, and height. We use the conversion 1 ft = 0.304 8 m and 100 cm = 1 m.

$$V = LWH$$
$$= (50.0 \text{ ft})\left(\frac{0.304\,8 \text{ m}}{1 \text{ ft}}\right)\times(26 \text{ ft})\left(\frac{0.304\,8 \text{ m}}{1 \text{ ft}}\right)\times(8.0 \text{ ft})\left(\frac{0.304\,8 \text{ m}}{1 \text{ ft}}\right)$$
$$= 294.5 \text{ m}^3 = \boxed{290 \text{ m}^3}$$
$$= (294.5 \text{ m}^3)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = \boxed{2.9\times10^8 \text{ cm}^3}$$

Both the 26-ft width and 8.0-ft height of the house have two significant figures, which is why our answer was rounded to 290 m^3.

P1.25 The aluminum sphere must be larger in volume to compensate for its lower density. We require equal masses:

$$m_{Al} = m_{Fe} \qquad \text{or} \qquad \rho_{Al} V_{Al} = \rho_{Fe} V_{Fe}$$

then use the volume of a sphere. By substitution,

$$\rho_{Al}\left(\frac{4}{3}\pi r_{Al}^{\ 3}\right) = \rho_{Fe}\left(\frac{4}{3}\pi (2.00\text{ cm})^3\right)$$

Now solving for the unknown,

$$r_{Al}^3 = \left(\frac{\rho_{Fe}}{\rho_{Al}}\right)(2.00\text{ cm})^3 = \left(\frac{7.86\times 10^3\text{ kg/m}^3}{2.70\times 10^3\text{ kg/m}^3}\right)(2.00\text{ cm})^3$$
$$= 23.3\text{ cm}^3$$

Taking the cube root, $\boxed{r_{Al} = 2.86\text{ cm}}$.

The aluminum sphere is 43% larger than the iron one in radius, diameter, and circumference. Volume is proportional to the cube of the linear dimension, so this excess in linear size gives it the (1.43)(1.43)(1.43) = 2.92 times larger volume it needs for equal mass.

P1.26 The mass of each sphere is $m_{Al} = \rho_{Al} V_{Al} = \dfrac{4\pi\rho_{Al} r_{Al}^3}{3}$

and $m_{Fe} = \rho_{Fe} V_{Fe} = \dfrac{4\pi\rho_{Fe} r_{Fe}^3}{3}$. Setting these masses equal,

$$\frac{4}{3}\pi\rho_{Al} r_{Al}^3 = \frac{4}{3}\pi\rho_{Fe} r_{Fe}^3 \rightarrow r_{Al} = r_{Fe}\sqrt[3]{\frac{\rho_{Fe}}{\rho_{Al}}}$$

$$r_{Al} = r_{Fe}\sqrt[3]{\frac{7.86}{2.70}} = \boxed{r_{Fe}(1.43)}$$

The resulting expression shows that the radius of the aluminum sphere is directly proportional to the radius of the balancing iron sphere. The aluminum sphere is 43% larger than the iron one in radius, diameter, and circumference. Volume is proportional to the cube of the linear dimension, so this excess in linear size gives it the $(1.43)^3$ = 2.92 times larger volume it needs for equal mass.

P1.27 We assume the paint keeps the same volume in the can and on the wall, and model the film on the wall as a rectangular solid, with its volume given by its "footprint" area, which is the area of the wall, multiplied by its thickness t perpendicular to this area and assumed to be uniform. Then,

$$V = At \quad \text{gives} \quad t = \frac{V}{A} = \frac{3.78 \times 10^{-3}\ \text{m}^3}{25.0\ \text{m}^2} = \boxed{1.51 \times 10^{-4}\ \text{m}}$$

The thickness of 1.5 tenths of a millimeter is comparable to the thickness of a sheet of paper, so this answer is reasonable. The film is many molecules thick.

P1.28 (a) To obtain the volume, we multiply the length, width, and height of the room, and use the conversion 1 m = 3.281 ft.

$$\begin{aligned} V &= (40.0\ \text{m})(20.0\ \text{m})(12.0\ \text{m}) \\ &= \left(9.60\times 10^3\ \text{m}^3\right)\left(\frac{3.281\ \text{ft}}{1\ \text{m}}\right)^3 \\ &= \boxed{3.39\times 10^5\ \text{ft}^3} \end{aligned}$$

(b) The mass of the air is

$$m = \rho_{\text{air}} V = \left(1.20\ \text{kg/m}^3\right)\left(9.60\times 10^3\ \text{m}^3\right) = 1.15\times 10^4\ \text{kg}$$

The student must look up the definition of weight in the index to find

$$F_g = mg = \left(1.15\times 10^4\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = 1.13\times 10^5\ \text{N}$$

where the unit of N of force (weight) is newtons.

Converting newtons to pounds,

$$F_g = (1.13\times 10^5\ \text{N})\left(\frac{1\ \text{lb}}{4.448\ \text{N}}\right) = \boxed{2.54\times 10^4\ \text{lb}}$$

P1.29 (a) The time interval required to repay the debt will be calculated by dividing the total debt by the rate at which it is repaid.

$$T = \frac{\$16\ \text{trillion}}{\$1000\,/\,\text{s}} = \frac{\$16 \times 10^{12}}{(\$1000\,/\,\text{s})\left(3.156 \times 10^7\ \text{s/yr}\right)} = \boxed{507\ \text{yr}}$$

(b) The number of bills is the distance to the Moon divided by the length of a dollar.

$$N = \frac{D}{\ell} = \frac{3.84\times 10^8\ \text{m}}{0.155\ \text{m}} = \boxed{2.48\times 10^9\ \text{bills}}$$

Sixteen trillion dollars is larger than this two-and-a-half billion dollars by more than six thousand times. The ribbon of bills comprising the debt reaches across the cosmic gulf thousands of times. Similar calculations show that the bills could span the distance between the Earth and the Sun sixteen times. The strip could encircle the Earth's equator nearly 62 000 times. With successive turns wound edge to edge without overlapping, the dollars would cover a zone centered on the equator and about 4.2 km wide.

P1.30 (a) To find the scale size of the nucleus, we multiply by the scaling factor

$$\begin{aligned} d_{\text{nucleus, scale}} &= d_{\text{nucleus, real}}\left(\frac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right) \\ &= \left(2.40\times 10^{-15}\text{ m}\right)\left(\frac{300\text{ ft}}{1.06\times 10^{-10}\text{ m}}\right) \\ &= 6.79\times 10^{-3}\text{ ft} \end{aligned}$$

or

$$d_{\text{nucleus, scale}} = \left(6.79\times 10^{-3}\text{ ft}\right)\left(\frac{304.8\text{ mm}}{1\text{ ft}}\right) = \boxed{2.07\text{ mm}}$$

(b) The ratio of volumes is simply the ratio of the cubes of the radii:

$$\frac{V_{\text{atom}}}{V_{\text{nucleus}}} = \frac{4\pi r_{\text{atom}}^3/3}{4\pi r_{\text{nucleus}}^3/3} = \left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3$$

$$= \left(\frac{1.06\times 10^{-10}\text{ m}}{2.40\times 10^{-15}\text{ m}}\right)^3 = \boxed{8.62\times 10^{13}\text{ times as large}}$$

Section 1.5 Estimates and Order-of-Magnitude Calculations

P1.31 Since we are only asked to find an estimate, we do not need to be too concerned about how the balls are arranged. Therefore, to find the number of balls we can simply divide the volume of an average-size living room (perhaps 15 ft × 20 ft × 8 ft) by the volume of an individual Ping-Pong ball. Using the approximate conversion 1 ft = 30 cm, we find

$$V_{\text{Room}} = (15\text{ ft})(20\text{ ft})(8\text{ ft})(30\text{ cm/ft})^3 \approx 6\times 10^7\text{ cm}^3$$

A Ping-Pong ball has a diameter of about 3 cm, so we can estimate its volume as a cube:

$$V_{\text{ball}} = (3\text{ cm})(3\text{ cm})(3\text{ cm}) \approx 30\text{ cm}^3$$

The number of Ping-Pong balls that can fill the room is

$$N \approx \frac{V_{\text{Room}}}{V_{\text{ball}}} \approx 2 \times 10^6 \text{ balls} \sim \boxed{10^6 \text{ balls}}$$

So a typical room can hold on the order of a million Ping-Pong balls. As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2} = 0.74$, so that at least 26% of the space will be empty.

P1.32 (a) We estimate the mass of the water in the bathtub. Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V = (0.5)(1.3)(0.5)(0.3) = 0.10 \text{ m}^3$$

The mass of this volume of water is

$$m_{\text{water}} = \rho_{\text{water}} V = \left(1\ 000 \text{ kg/m}^3\right)\left(0.10 \text{ m}^3\right) = 100 \text{ kg} \boxed{\sim 10^2 \text{ kg}}$$

(b) Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}} = \rho_{\text{copper}} V = \left(8\ 920 \text{ kg/m}^3\right)\left(0.10 \text{ m}^3\right) = 892 \text{ kg} \boxed{\sim 10^3 \text{ kg}}$$

P1.33 Don't reach for the telephone book or do a Google search! Think. Each full-time piano tuner must keep busy enough to earn a living. Assume a total population of 10^7 people. Also, let us estimate that one person in one hundred owns a piano. Assume that in one year a single piano tuner can service about 1 000 pianos (about 4 per day for 250 weekdays), and that each piano is tuned once per year.

Therefore, the number of tuners

$$= \left(\frac{1 \text{ tuner}}{1\ 000 \text{ pianos}}\right)\left(\frac{1 \text{ piano}}{100 \text{ people}}\right)\left(10^7 \text{ people}\right) \sim \boxed{100 \text{ tuners}}$$

If you did reach for an Internet directory, you would have to count. Instead, have faith in your estimate. Fermi's own ability in making an order-of-magnitude estimate is exemplified by his measurement of the energy output of the first nuclear bomb (the Trinity test at Alamogordo, New Mexico) by observing the fall of bits of paper as the blast wave swept past his station, 14 km away from ground zero.

P1.34 A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make

$$(50\ 000 \text{ mi})(5\ 280 \text{ ft/mi})(1 \text{ rev}/8 \text{ ft}) = 3 \times 10^7 \text{ rev} \sim \boxed{10^7 \text{ rev}}$$

Section 1.6 Significant Figures

P1.35 We will use two different methods to determine the area of the plate and the uncertainty in our answer.

METHOD ONE: We treat the best value with its uncertainty as a binomial, (21.3 ± 0.2) cm × (9.8 ± 0.1) cm, and obtain the area by expanding:

$$A = \left[21.3(9.8) \pm 21.3(0.1) \pm 0.2(9.8) \pm (0.2)(0.1)\right]\text{cm}^2$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A = \boxed{209\ \text{cm}^2 \pm 4\ \text{cm}^2}$$

METHOD TWO: We add the fractional uncertainties in the data.

$$A = (21.3\ \text{cm})(9.8\ \text{cm}) \pm \left(\frac{0.2}{21.3} + \frac{0.1}{9.8}\right)$$

$$= 209\ \text{cm}^2 \pm 2\% = 209\ \text{cm}^2 \pm 4\ \text{cm}^2$$

P1.36 (a) The ± 0.2 following the 78.9 expresses uncertainty in the last digit. Therefore, there are $\boxed{\textbf{three}}$ significant figures in 78.9 ± 0.2.

(b) Scientific notation is often used to remove the ambiguity of the number of significant figures in a number. Therefore, all the digits in 3.788 are significant, and 3.788×10^9 has $\boxed{\textbf{four}}$ significant figures.

(c) Similarly, 2.46 has three significant figures, therefore 2.46×10^{-6} has $\boxed{\textbf{three}}$ significant figures.

(d) Zeros used to position the decimal point are not significant. Therefore 0.005 3 has $\boxed{\textbf{two}}$ significant figures.

Uncertainty in a measurement can be the result of a number of factors, including the skill of the person doing the measurements, the precision and the quality of the instrument used, and the number of measurements made.

P1.37 We work to nine significant digits:

$$1\ \text{yr} = 1\ \text{yr}\left(\frac{365.\ 242\ 199\ \text{d}}{1\ \text{yr}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \boxed{315\ 569\ 26.0\ \text{s}}$$

P1.38 (a) $756 + 37.2 + 0.83 + 2 = 796.03 \to \boxed{796}$, since the number with the fewest decimal places is 2.

(b) $(0.003\ 2)\{2 \text{ s.f.}\}\times(356.3)\{4 \text{ s.f.}\} = 1.140\ 16 = \{2 \text{ s.f.}\}\ \boxed{1.1}$

(c) $5.620\{4 \text{ s.f.}\}\times\pi\{>4 \text{ s.f.}\} = 17.656 = \{4 \text{ s.f.}\}\ \boxed{17.66}$

P1.39 Let o represent the number of ordinary cars and s the number of sport utility vehicles. We know $o = s + 0.947s = 1.947s$, and $o = s + 18$.

We eliminate o by substitution:

$$s + 18 = 1.947s \rightarrow 0.947s = 18 \rightarrow s = 18 / 0.947 = \boxed{19}$$

P1.40 "One and one-third months" = 4/3 months. Treat this problem as a conversion:

$$\left(\frac{1 \text{ bar}}{4/3 \text{ months}}\right)\left(\frac{12 \text{ months}}{1 \text{ year}}\right) = \boxed{9 \text{ bars/year}}$$

P1.41 The tax amount is \$1.36 – \$1.25 = \$0.11. The tax rate is

$$\$0.11/\$1.25 = 0.0880 = \boxed{8.80\%}$$

P1.42 We are given the ratio of the masses and radii of the planets Uranus and Neptune:

$$\frac{M_\text{N}}{M_\text{U}} = 1.19, \text{ and } \frac{r_\text{N}}{r_\text{U}} = 0.969$$

The definition of density is $\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{V}$, where $V = \frac{4}{3}\pi r^3$ for a sphere, and we assume the planets have a spherical shape.

We know $\rho_\text{U} = 1.27\times10^3 \text{ kg/m}^3$. Compare densities:

$$\frac{\rho_\text{N}}{\rho_\text{U}} = \frac{M_\text{N}/V_\text{N}}{M_\text{U}/V_\text{U}} = \left(\frac{M_\text{N}}{M_\text{U}}\right)\left(\frac{V_\text{U}}{V_\text{N}}\right) = \left(\frac{M_\text{N}}{M_\text{U}}\right)\left(\frac{r_\text{U}}{r_\text{N}}\right)^3$$

$$= (1.19)\left(\frac{1}{0.969}\right)^3 = 1.307\ 9$$

which gives

$$\rho_\text{N} = (1.3079)\left(1.27\times10^3 \text{ kg/m}^3\right) = \boxed{1.66\times10^3 \text{ kg/m}^3}$$

P1.43 Let s represent the number of sparrows and m the number of more interesting birds. We know $s/m = 2.25$ and $s + m = 91$.

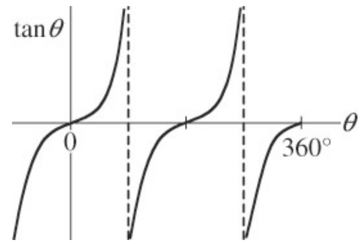
We eliminate m by substitution:

$$m = s/2.25 \rightarrow s + s/2.25 = 91 \rightarrow 1.444s = 91$$

$$\rightarrow s = 91/1.444 = \boxed{63}$$

P1.44 We require

$$\sin\theta = -3\cos\theta, \text{ or } \frac{\sin\theta}{\cos\theta} = \tan\theta = -3$$

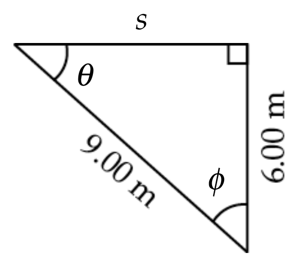


ANS. FIG. P1.44

For $\tan^{-1}(-3) = \arctan(-3)$, your calculator may return –71.6°, but this angle is not between 0° and 360° as the problem requires. The tangent function is negative in the second quadrant (between 90° and 180°) and in the fourth quadrant (from 270° to 360°). The solutions to the equation are then

$$360° - 71.6° = \boxed{288°} \text{ and } 180° - 71.6 = \boxed{108°}$$

***P1.45** (a) ANS. FIG. P1.45 shows that the hypotenuse of the right triangle has a length of 9.00 m and the unknown side is opposite the angle ϕ. Since the two angles in the triangle are not known, we can obtain the length of the unknown side, which we will represent as s, using the Pythagorean Theorem:



ANS. FIG. P1.45

$$s^2 + (6.00\text{ m})^2 = (9.00\text{ m})^2$$
$$s^2 = (9.00\text{ m})^2 - (6.00\text{ m})^2 = 45$$

which gives $s = \boxed{6.71\text{ m}}$. We express all of our answers in three significant figures since the lengths of the two known sides of the triangle are given with three significant figures.

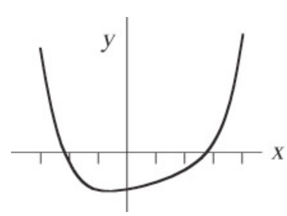
(b) From ANS. FIG. P1.45, the tangent of θ is equal to ratio of the side opposite the angle, 6.00 m in length, and the side adjacent to the angle, s = 6.71 m, and is given by

$$\tan\theta = \frac{6.00\text{ m}}{s} = \frac{6.00\text{ m}}{6.71\text{ m}} = \boxed{0.894}$$

(c) From ANS. FIG. P1.45, the sine of ϕ is equal to ratio of the side opposite the angle, s = 6.71 m, and the hypotenuse of the triangle, 9.00 m in length, and is given by

$$\sin\phi = \frac{s}{9.00\text{ m}} = \frac{6.71\text{ m}}{9.00\text{ m}} = \boxed{0.745}$$

P1.46 For those who are not familiar with solving equations numerically, we provide a detailed solution. It goes beyond proving that the suggested answer works.



ANS. FIG. P1.46

The equation $2x^4 - 3x^3 + 5x - 70 = 0$ is quartic, so we do not attempt to solve it with algebra. To find how many real solutions the equation has and to estimate them, we graph the expression:

x	–3	–2	–1	0	1	2	3	4
$y = 2x^4 - 3x^3 + 5x - 70$	158	–24	–70	–70	–66	–52	26	270

We see that the equation $y = 0$ has two roots, one around $x = -2.2$ and the other near $x = +2.7$. To home in on the first of these solutions we compute in sequence:

When $x = -2.2$, $y = -2.20$. The root must be between $x = -2.2$ and $x = -3$.
When $x = -2.3$, $y = 11.0$. The root is between $x = -2.2$ and $x = -2.3$.
When $x = -2.23$, $y = 1.58$. The root is between $x = -2.20$ and $x = -2.23$.
When $x = -2.22$, $y = 0.301$. The root is between $x = -2.20$ and -2.22.
When $x = -2.215$, $y = -0.331$. The root is between $x = -2.215$ and -2.22.
We could next try $x = -2.218$, but we already know to three-digit precision that the root is $x = -2.22$.

P1.47 When the length changes by 15.8%, the mass changes by a much larger percentage. We will write each of the sentences in the problem as a mathematical equation.

Mass is proportional to length cubed: $m = k\ell^3$, where k is a constant. This model of growth is reasonable because the lamb gets thicker as it gets longer, growing in three-dimensional space.

At the initial and final points, $m_i = k\ell_i^3$ and $m_f = k\ell_f^3$

Length changes by 15.8%: 15.8% **of** ℓ means 0.158 **times** ℓ.

Thus $\ell_i + 0.158\,\ell_i = \ell_f$ and $\ell_f = 1.158\,\ell_i$

Mass increases by 17.3 kg: $m_i + 17.3\text{ kg} = m_f$

Now we combine the equations using algebra, eliminating the unknowns ℓ_i, ℓ_f, k, and m_i by substitution:

From $\ell_f = 1.158\,\ell_i$, we have $\ell_f^3 = 1.158^3\,\ell_i^3 = 1.553\,\ell_i^3$

Then

$$m_f = k\ell_f^3 = k(1.553)\ell_i^3 = 1.553\,k\ell_i^3 = 1.553 m_i \quad \text{and} \quad m_i = m_f/1.553$$

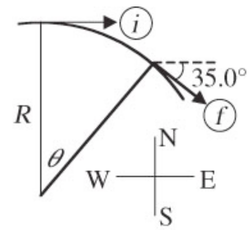
Next,

$$m_i + 17.3\text{ kg} = m_f \quad \text{becomes} \quad m_f/1.553 + 17.3\text{ kg} = m_f$$

Solving, $17.3\text{ kg} = m_f - m_f/1.553 = m_f(1 - 1/1.553) = 0.356\,m_f$

and $$m_f = \frac{17.3\text{ kg}}{0.356} = \boxed{48.6\text{ kg}}.$$

P1.48 We draw the radius to the initial point and the radius to the final point. The angle θ between these two radii has its sides perpendicular, right side to right side and left side to left side, to the 35° angle between the original and final tangential directions of travel. A most useful theorem from geometry then identifies these angles as equal: $\theta = 35°$. The whole circumference of a 360° circle of the same radius is $2\pi R$. By proportion, then



ANS. FIG. P1.48

$$\frac{2\pi R}{360°} = \frac{840\text{ m}}{35°}$$

$$R = \left(\frac{360°}{2\pi}\right)\left(\frac{840\text{ m}}{35°}\right) = \frac{840\text{ m}}{0.611} = \boxed{1.38 \times 10^3\text{ m}}$$

We could equally well say that the measure of the angle in radians is

$$\theta = 35° = 35°\left(\frac{2\pi\text{ radians}}{360°}\right) = 0.611\text{ rad} = \frac{840\text{ m}}{R}$$

Solving yields $R = 1.38$ km.

P1.49 Use substitution to solve simultaneous equations. We substitute $p = 3q$ into each of the other two equations to eliminate p:

$$\begin{cases} 3qr = qs \\ \frac{1}{2}3qr^2 + \frac{1}{2}qs^2 = \frac{1}{2}qt^2 \end{cases}$$

These simplify to $\begin{cases} 3r = s \\ 3r^2 + s^2 = t^2 \end{cases}$, assuming $q \neq 0$.

We substitute the upper relation into the lower equation to eliminate s:

$$3r^2 + (3r)^2 = t^2 \rightarrow 12r^2 = t^2 \rightarrow \frac{t^2}{r^2} = 12$$

We now have the ratio of t to r: $\boxed{\frac{t}{r} = \pm\sqrt{12} = \pm 3.46}$

P1.50 First, solve the given equation for Δt:

$$\Delta t = \frac{4QL}{k\pi d^2(T_h - T_c)} = \left[\frac{4QL}{k\pi(T_h - T_c)}\right]\left[\frac{1}{d^2}\right]$$

(a) Making d three times larger with d^2 in the bottom of the fraction makes Δt $\boxed{\text{nine times smaller}}$.

(b) $\boxed{\Delta t \text{ is inversely proportional to the square of } d.}$

(c) $\boxed{\text{Plot } \Delta t \text{ on the vertical axis and } 1/d^2 \text{ on the horizontal axis.}}$

(d) From the last version of the equation, the slope is $\boxed{4QL / k\pi\left(T_h - T_c\right)}$. Note that this quantity is constant as both Δt and d vary.

P1.51 (a) The fourth experimental point from the top is a circle: this point lies just above the best-fit curve that passes through the point (400 cm^2, 0.20 g). The interval between horizontal grid lines is 1 space = 0.05 g. We estimate from the graph that the circle has a vertical separation of 0.3 spaces = $\boxed{0.015 \text{ g}}$ above the best-fit curve.

(b) The best-fit curve passes through 0.20 g:

$$\left(\frac{0.015 \text{ g}}{0.20 \text{ g}}\right)\times 100 = \boxed{8\%}$$

(c) The best-fit curve passes through the origin and the point (600 cm^3, 3.1 g). Therefore, the slope of the best-fit curve is

$$\text{slope} = \left(\frac{3.1 \text{ g}}{600 \text{ cm}^3}\right) = \boxed{5.2\times 10^{-3} \frac{\text{g}}{\text{cm}^3}}$$

(d) $\boxed{\text{For shapes cut from this copy paper, the mass of the cutout is proportional to its area. The proportionality constant is } 5.2 \text{ g/m}^2 \pm 8\%\text{, where the uncertainty is estimated.}}$

(e) $\boxed{\text{This result is to be expected if the paper has thickness and density that are uniform within the experimental uncertainty.}}$

(f) $\boxed{\text{The slope is the areal density of the paper, its mass per unit area.}}$

P1.52 $r = (6.50 \pm 0.20) \text{ cm} = (6.50 \pm 0.20)\times 10^{-2} \text{ m}$

$$m = (1.85 + 0.02)\text{kg}$$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$

also, $$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} + \frac{3\delta r}{r}$$

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta\rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103,$$

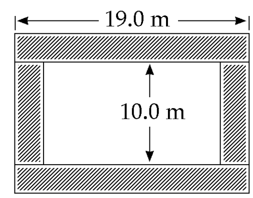
$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5\times10^{-2}\text{ m}\right)^3} = \boxed{1.61\times10^3\text{ kg/m}^3}$$

then $\delta\rho = 0.103\rho = \boxed{0.166\times10^3\text{ kg/m}^3}$

and $\rho \pm \delta\rho = \boxed{(1.61\pm0.17)\times10^3\text{ kg/m}^3} = (1.6\pm0.2)\times10^3\text{ kg/m}^3.$

***P1.53** The volume of concrete needed is the sum of the four sides of sidewalk, or

$$V = 2V_1 + 2V_2 = 2(V_1 + V_2)$$



ANS. FIG. P1.53

The figure on the right gives the dimensions needed to determine the volume of each portion of sidewalk:

$$V_1 = (17.0\text{ m} + 1.0\text{ m} + 1.0\text{ m})(1.0\text{ m})(0.09\text{ m}) = 1.70\text{ m}^3$$

$$V_2 = (10.0\text{ m})(1.0\text{ m})(0.090\text{ m}) = 0.900\text{ m}^3$$

$$V = 2\left(1.70\text{ m}^3 + 0.900\text{ m}^3\right) = \boxed{5.2\text{ m}^3}$$

The uncertainty in the volume is the sum of the uncertainties in each dimension:

$$\left.\begin{aligned}\frac{\delta\ell_1}{\ell_1} &= \frac{0.12\text{ m}}{19.0\text{ m}} = 0.0063\\ \frac{\delta w_1}{w_1} &= \frac{0.01\text{ m}}{1.0\text{ m}} = 0.010\\ \frac{\delta t_1}{t_1} &= \frac{0.1\text{ cm}}{9.0\text{ cm}} = 0.011\end{aligned}\right\} \frac{\delta V}{V} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{3\%}$$

Additional Problems

P1.54 (a) Let d represent the diameter of the coin and h its thickness. The gold plating is a layer of thickness t on the surface of the coin; so, the mass of the gold is

$$m = \rho V = \rho\left[2\pi\frac{d^2}{4} + \pi dh\right]t$$

$$= \left(19.3\frac{\text{g}}{\text{cm}^3}\right)\left[2\pi\frac{(2.41\text{ cm})^2}{4} + \pi(2.41\text{ cm})(0.178\text{ cm})\right]$$

$$\times\left(1.8\times10^{-7}\text{ m}\right)\left(\frac{10^2\text{ cm}}{1\text{ m}}\right)$$

$$= 0.003\ 64\text{ g}$$

and the cost of the gold added to the coin is

$$\text{cost} = \left(0.003\ 64\text{ g}\right)\left(\frac{\$10}{1\text{ g}}\right) = \$0.036\ 4 = \boxed{3.64\text{ cents}}$$

(b) The cost is negligible compared to \$4.98.

P1.55 It is desired to find the distance x such that

$$\frac{x}{100\text{ m}} = \frac{1\ 000\text{ m}}{x}$$

(i.e., such that x is the same multiple of 100 m as the multiple that 1 000 m is of x). Thus, it is seen that

$$x^2 = (100\text{ m})(1\ 000\text{ m}) = 1.00\times10^5\text{ m}^2$$

and therefore

$$x = \sqrt{1.00\times10^5\text{ m}^2} = \boxed{316\text{ m}}$$

P1.56 (a) A Google search yields the following dimensions of the intestinal tract:

small intestines: length ≅ 20 ft ≅ 6 m, diameter ≅ 1.5 in ≅ 4 cm

large intestines: length ≅ 5 ft ≅ 1.5 m, diameter ≅ 2.5 in ≅ 6 cm

Treat the intestines as two cylinders: the volume of a cylinder of diameter d and length L is $V = \frac{\pi}{4}d^2L$.

The volume of the intestinal tract is

$$V = V_{\text{small}} + V_{\text{large}}$$

$$V = \frac{\pi}{4}(0.04\,\text{m})^2(6\,\text{m}) + \frac{\pi}{4}(0.06\,\text{m})^2(1.5\,\text{m})$$
$$= 0.0117\,\text{m}^3 \cong 10^{-2}\,\text{m}^3$$

Assuming 1% of this volume is occupied by bacteria, the volume of bacteria is

$$V_{\text{bac}} = (10^{-2}\ \text{m}^3)(0.01) = 10^{-4}\ \text{m}^3$$

Treating a bacterium as a cube of side $L = 10^{-6}$ m, the volume of one bacterium is about $L^3 = 10^{-18}$ m^3. The number of bacteria in the intestinal tract is about

$$(10^{-4}\ \text{m}^3)\left(\frac{1\ \text{bacterium}}{10^{-18}\ \text{m}^3}\right) = \boxed{10^{14}\ \text{bacteria!}}$$

(b) The large number of bacteria suggests they must be $\boxed{\text{beneficial}}$, otherwise the body would have developed methods a long time ago to reduce their number. It is well known that certain types of bacteria in the intestinal tract are beneficial: they aid digestion, as well as prevent dangerous bacteria from flourishing in the intestines.

P1.57 We simply multiply the distance between the two galaxies by the scale factor used for the dinner plates. The scale factor used in the "dinner plate" model is

$$S = \left(\frac{0.25\ \text{m}}{1.0\times 10^5\ \text{light-years}}\right) = 2.5\times 10^{-6}\ \text{m/ly}$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}}S = (2.0\times 10^6\ \text{ly})(2.5\times 10^{-6}\ \text{m/ly}) = \boxed{5.0\ \text{m}}$$

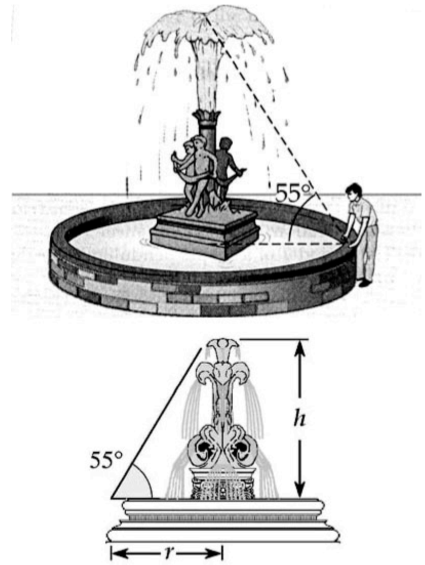
P1.58 Assume the winner counts one dollar per second, and the winner tries to maintain the count without stopping. The time interval required for the task would be

$$\$10^6\left(\frac{1\ \text{s}}{\$1}\right)\left(\frac{1\ \text{hour}}{3600\ \text{s}}\right)\left(\frac{1\ \text{work week}}{40\ \text{hours}}\right) = 6.9\ \text{work weeks.}$$

The scenario has the contestants succeeding on the whole. But the calculation shows that is impossible. It just takes too long!

P1.59 We imagine a top view to figure the radius of the pool from its circumference. We imagine a straight-on side view to use trigonometry to find the height.

Define a right triangle whose legs represent the height and radius of the fountain. From the dimensions of the fountain and the triangle, the circumference is $C = 2\pi r$ and the angle satisfies $\tan\phi = h/r$.



ANS. FIG. P1.59

Then by substitution

$$h = r\tan\phi = \left(\frac{C}{2\pi}\right)\tan\phi$$

Evaluating,

$$h = \left(\frac{15.0\text{ m}}{2\pi}\right)\tan 55.0° = \boxed{3.41\text{ m}}$$

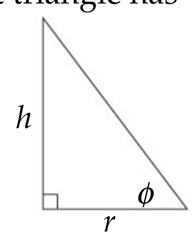
When we look at a three-dimensional system from a particular direction, we may discover a view to which simple mathematics applies.

P1.60 The fountain has height *h*; the pool has circumference *C* with radius *r*. The figure shows the geometry of the problem: a right triangle has base *r*, height *h*, and angle ϕ. From the triangle,

$$\tan\phi = h/r$$

We can find the radius of the circle from its circumference, $C = 2\pi r$, and then solve for the height using

$$\boxed{h = r\tan\phi = (\tan\phi)C/2\pi}$$



ANS. FIG. P1.60

P1.61 The density of each material is $\rho = \dfrac{m}{v} = \dfrac{m}{\pi r^2 h} = \dfrac{4m}{\pi D^2 h}$.

$$\text{Al: } \rho = \frac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})} = \boxed{2.75\frac{\text{g}}{\text{cm}^3}\text{; this is 2\% larger}}$$

than the tabulated value, 2.70 g/cm^3.

$$\text{Cu: } \rho = \frac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})} = \boxed{9.36\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 8.92 g/cm^3.

$$\text{brass: } \rho = \frac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})} = \boxed{8.91\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 8.47 g/cm³.

$$\text{Sn: } \rho = \frac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})} = \boxed{7.68\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 7.31 g/cm³.

$$\text{Fe: } \rho = \frac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})} = \boxed{7.88\frac{\text{g}}{\text{cm}^3}\text{; this is 0.3\% larger}}$$

than the tabulated value, 7.86 g/cm³.

P1.62 The volume of the galaxy is

$$\pi r^2 t = \pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right) \sim 10^{61}\text{ m}^3$$

If the distance between stars is 4×10^{16}, then there is one star in a volume on the order of

$$\left(4\times 10^{16}\text{ m}\right)^3 \sim 10^{50}\text{ m}^3$$

The number of stars is about $\dfrac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}} \sim \boxed{10^{11}\text{ stars}}$.

P1.63 We define an average national fuel consumption rate based upon the total miles driven by all cars combined. In symbols,

$$\text{fuel consumed} = \frac{\text{total miles driven}}{\text{average fuel consumption rate}}$$

or

$$f = \frac{s}{c}$$

For the current rate of 20 mi/gallon we have

$$f = \frac{\left(100 \times 10^6\text{ cars}\right)\left(10^4\text{ (mi/yr)/car}\right)}{20\text{ mi/gal}} = 5 \times 10^{10}\text{ gal/yr}$$

Since we consider the same total number of miles driven in each case, at 25 mi/gal we have

$$f = \frac{\left(100 \times 10^6\text{ cars}\right)\left(10^4\text{ (mi/yr)/car}\right)}{25\text{ mi/gal}} = 4 \times 10^{10}\text{ gal/yr}$$

Thus we estimate a change in fuel consumption of

$$\Delta f = 4\times 10^{10}\text{ gal/yr} - 5\times 10^{10}\text{ gal/yr} = \boxed{-1\times 10^{10}\text{ gal/yr}}$$

The negative sign indicates that the change is a reduction. It is a fuel savings of ten billion gallons each year.

P1.64 (a) The mass is equal to the mass of a sphere of radius 2.6 cm and density 4.7 g/cm^3, minus the mass of a sphere of radius a and density 4.7 g/cm^3, plus the mass of a sphere of radius a and density 1.23 g/cm^3.

$$m = \rho_1\left(\frac{4}{3}\pi r^3\right) - \rho_1\left(\frac{4}{3}\pi a^3\right) + \rho_2\left(\frac{4}{3}\pi a^3\right)$$

$$= \left(\frac{4}{3}\pi\right)\left[\left(4.7\ \text{g/cm}^3\right)\left(2.6\ \text{cm}\right)^3 - \left(4.7\ \text{g/cm}^3\right)a^3 + \left(1.23\ \text{g/cm}^3\right)a^3\right]$$

$$m = \boxed{346\ \text{g} - \left(14.5\ \text{g/cm}^3\right)a^3}$$

(b) The mass is maximum for $\boxed{a = 0}$.

(c) $\boxed{346\ \text{g}}$.

(d) $\boxed{\text{Yes}}$. This is the mass of the uniform sphere we considered in the first term of the calculation.

(e) $\boxed{\text{No change, so long as the wall of the shell is unbroken.}}$

P1.65 Answers may vary depending on assumptions:

typical length of bacterium: $L = 10^{-6}$ m

typical volume of bacterium: $L^3 = 10^{-18}\ \text{m}^3$

surface area of Earth: $A = 4\pi r^2 = 4\pi\left(6.38\times 10^6\ \text{m}\right)^2 = 5.12\times 10^{14}\,\text{m}^2$

(a) If we assume the bacteria are found to a depth d = 1000 m below Earth's surface, the volume of Earth containing bacteria is about

$$V = \left(4\pi r^2\right)d = 5.12\times 10^{17}\ \text{m}^3$$

If we assume an average of 1000 bacteria in every 1 mm^3 of volume, then the number of bacteria is

$$\left(\frac{1000\ \text{bacteria}}{1\ \text{mm}^3}\right)\left(\frac{10^3\ \text{mm}}{1\ \text{m}}\right)^3 (5.12\times 10^{17}\ \text{m}^3) \approx \boxed{5.12\times 10^{29}\ \text{bacteria}}$$

(b) Assuming a bacterium is basically composed of water, the total mass is

$$\left(10^{29}\ \text{bacteria}\right)\left(\frac{10^{-18}\ \text{m}^3}{1\ \text{bacterium}}\right)\left(\frac{10^3\ \text{kg}}{1\ \text{m}^3}\right) = \boxed{10^{14}\ \text{kg}}$$

P1.66 The rate of volume increase is

$$\frac{dV}{dt}=\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}=\left(4\pi r^2\right)\frac{dr}{dt}$$

(a) $\frac{dV}{dt}=4\pi(6.5\text{ cm})^2(0.9\text{ cm/s})=\boxed{478\text{ cm}^3/\text{s}}$

(b) The rate of increase of the balloon's radius is

$$\frac{dr}{dt}=\frac{dV/dt}{4\pi r^2}=\frac{478\text{ cm}^3/\text{s}}{4\pi(13\text{ cm})^2}=\boxed{0.225\text{ cm/s}}$$

(c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.67 (a) We have $B + C(0) = 2.70\text{ g/cm}^3$ and $B + C(14\text{ cm}) = 19.3\text{ g/cm}^3$.

We know $\boxed{B=2.70\text{ g/cm}^3}$, and we solve for C by subtracting:

$C(14\text{ cm}) = 19.3\text{ g/cm}^3 - B = 16.6\text{ g/cm}^3$, so $\boxed{C=1.19\text{ g/cm}^4}$

(b) The integral is

$$m=(9.00\text{ cm}^2)\int_0^{14\text{ cm}}(B+Cx)\,dx$$

$$=(9.00\text{ cm}^2)\left(Bx+\frac{C}{2}x^2\right)\Bigg|_0^{14\text{ cm}}$$

$$m=(9.00\text{ cm}^2)\Big\{\left(2.70\text{ g/cm}^3\right)(14\text{ cm}-0)$$

$$+\left(1.19\text{ g/cm}^4/2\right)\left[(14\text{ cm})^2-0\right]\Big\}$$

$$=340\text{ g}+1046\text{ g}=1390\text{ g}=\boxed{1.39\text{ kg}}$$

P1.68 The table below shows α in degrees, α in radians, $\tan(\alpha)$, and $\sin(\alpha)$ for angles from 15.0° to 31.1°:

α' (deg)	α (rad)	$\tan(\alpha)$	$\sin(\alpha)$	difference between α and $\tan\alpha$
15.0	0.262	0.268	0.259	2.30%
20.0	0.349	0.364	0.342	4.09%
30.0	0.524	0.577	0.500	9.32%
33.0	0.576	0.649	0.545	11.3%
31.0	0.541	0.601	0.515	9.95%
31.1	0.543	0.603	0.516	10.02%

We see that α in radians, $\tan(\alpha)$, and $\sin(\alpha)$ start out together from zero and diverge only slightly in value for small angles. Thus $\boxed{31.0°}$ is the largest angle for which $\dfrac{\tan\alpha-\alpha}{\tan\alpha}<0.1$.

P1.69 We write "millions of cubic feet" as $10^6\ \text{ft}^3$, and use the given units of time and volume to assign units to the equation.

$$V=(1.50\times10^6\,\text{ft}^3/\text{mo})t+(0.008\,00\times10^6\,\text{ft}^3/\text{mo}^2)t^2$$

To convert the units to seconds, use

$$1\text{ month}=(30.0\text{ d})\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)=2.59\times10^6\,\text{s}$$

to obtain

$$\begin{aligned}V&=\left(1.50\times10^6\,\frac{\text{ft}^3}{\text{mo}}\right)\left(\frac{1\text{ mo}}{2.59\times10^6\text{ s}}\right)t\\&\quad+\left(0.008\,00\times10^6\,\frac{\text{ft}^3}{\text{mo}^2}\right)\left(\frac{1\text{ mo}}{2.59\times10^6\text{ s}}\right)^2t^2\\&=(0.579\text{ ft}^3/\text{s})t+(1.19\times10^{-9}\text{ ft}^3/\text{s}^2)t^2\end{aligned}$$

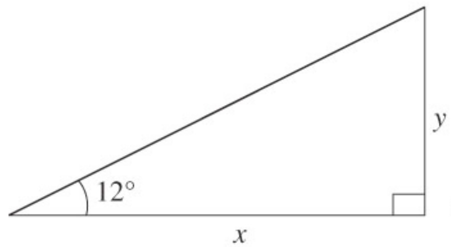
or

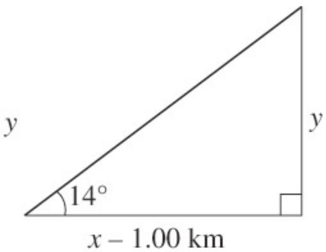
$$V=\boxed{0.579t+1.19\times10^{-9}t^2}$$

where V is in cubic feet and t is in seconds. The coefficient of the first term is the volume rate of flow of gas at the beginning of the month.

The second term's coefficient is related to how much the rate of flow increases every second.

P1.70 (a) and (b), the two triangles are shown.





ANS. FIG. P1.70(a) **ANS. FIG. P1.70(b)**

(c) From the triangles,

$$\tan 12.0° = \frac{y}{x} \rightarrow \boxed{y = x\tan 12.0°}$$

and $$\tan 14.0° = \frac{y}{(x - 1.00\text{ km})} \rightarrow \boxed{y = (x - 1.00\text{ km})\tan 14.0°}.$$

(d) Equating the two expressions for y, we solve to find $\boxed{y = 1.44\text{ km.}}$

P1.71 Observe in Fig. 1.71 that the radius of the horizontal cross section of the bottle is a relative maximum or minimum at the two radii cited in the problem; thus, we recognize that as the liquid level rises, the time rate of change of the diameter of the cross section will be zero at these positions.

The volume of a particular thin cross section of the shampoo of thickness h and area A is $V = Ah$, where $A = \pi r^2 = \pi D^2/4$. Differentiate the volume with respect to time:

$$\frac{dV}{dt} = A\frac{dh}{dt} + h\frac{dA}{dt} = A\frac{dh}{dt} + h\frac{d}{dt}(\pi r^2) = A\frac{dh}{dt} + 2\pi hr\frac{dr}{dt}$$

Because the radii given are a maximum and a minimum value, $dr/dt = 0$, so

$$\frac{dV}{dt} + A\frac{dh}{dt} = Av \rightarrow v = \frac{1}{A}\frac{dV}{dt} = \frac{1}{\pi D^2/4}\frac{dV}{dt} = \frac{4}{\pi D^2}\frac{dV}{dt}$$

where $v = dh/dt$ is the speed with which the level of the fluid rises.

(a) For $D = 6.30$ cm,

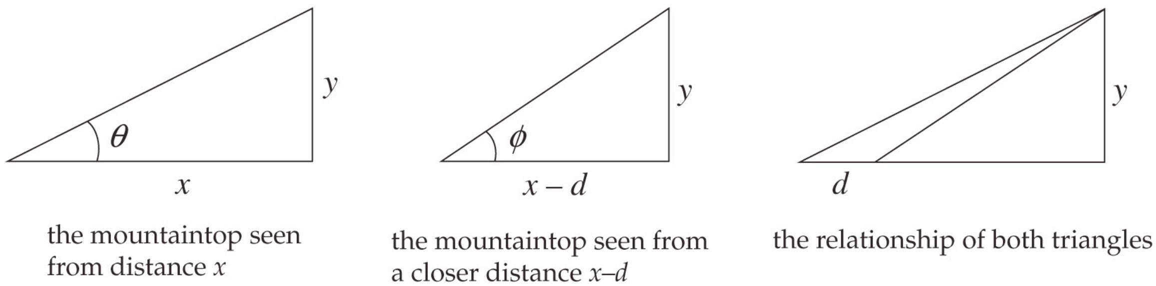
$$v = \frac{4}{\pi(6.30\text{ cm})^2}(16.5\text{ cm}^3/\text{s}) = \boxed{0.529\text{ cm/s}}$$

(b) For $D = 1.35$ cm,

$$v = \frac{4}{\pi(1.35\text{ cm})^2}(16.5\text{ cm}^3/\text{s}) = \boxed{11.5\text{ cm/s}}$$

Challenge Problems

P1.72 The geometry of the problem is shown below.



ANS. FIG. P1.72

From the triangles in ANS. FIG. P1.72,

$$\tan\theta = \frac{y}{x} \rightarrow y = x\tan\theta$$

and

$$\tan\phi = \frac{y}{x-d} \rightarrow y = (x-d)\tan\phi$$

Equate these two expressions for y and solve for x:

$$x\tan\theta = (x-d)\tan\phi \rightarrow d\tan\phi = x(\tan\phi - \tan\theta)$$

$$\rightarrow x = \frac{d\tan\phi}{\tan\phi - \tan\theta}$$

Take the expression for x and substitute it into either expression for y:

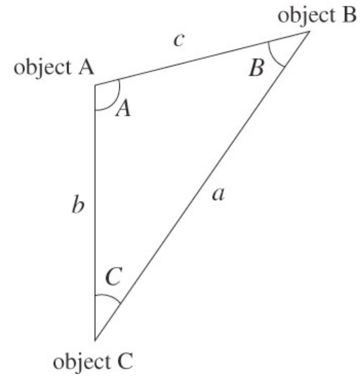
$$y = x\tan\theta = \boxed{\frac{d\tan\phi\tan\theta}{\tan\phi - \tan\theta}}$$

P1.73 The geometry of the problem suggests we use the law of cosines to relate known sides and angles of a triangle to the unknown sides and angles. Recall that the sides a, b, and c with opposite angles A, B, and C have the following relationships:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



ANS. FIG. P1.73

For the cows in the meadow, the triangle has sides a = 25.0 m and b = 15.0 m, and angle C = 20.0°, where object A = cow A, object B = cow B, and object C = you.

(a) Find side c:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (25.0\text{ m})^2 + (15.0\text{ m})^2 - 2(25.0\text{ m})(15.0\text{ m})\cos(20.0°)$$

$$c = \boxed{12.1\text{ m}}$$

(b) Find angle A:

$$a^2 = b^2 + c^2 - 2bc\cos A \rightarrow$$

$$\cos A = \frac{a^2 - b^2 - c^2}{2bc} = \frac{(25.0\text{ m})^2 - (15.0\text{ m})^2 - (12.1\text{ m})^2}{2(15.0\text{ m})(12.1\text{ m})}$$

$$\rightarrow A = 134.8° = \boxed{135°}$$

(c) Find angle B:

$$b^2 = c^2 + a^2 - 2ca\cos B \rightarrow$$

$$\cos B = \frac{b^2 - c^2 - a^2}{2ca} = \frac{(15.0\text{ m})^2 - (25.0\text{ m})^2 - (12.1\text{ m})^2}{2(25.0\text{ m})(12.1\text{ m})}$$

$$\rightarrow B = \boxed{25.2°}$$

(d) For the situation, object A = star A, object B = star B, and object C = our Sun (or Earth); so, the triangle has sides a = 25.0 ly, b = 15.0 ly, and angle C = 20.0°. The numbers are the same, except for units, as in part (b); thus, $\boxed{\text{angle } A = 135°.}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P1.2 2.15×10^4 kg/m^3

P1.4 (a) 2.3×10^{17} kg/m^3; (b) 1.0×10^{13} times the density of osmium

P1.6 $\dfrac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}$

P1.8 (a) $8.42\times10^{22}\ \dfrac{\text{Cu-atom}}{\text{cm}^3}$; (b) 1.19×10^{-23} cm^3/Cu-atom;
(c) 2.28×10^{-8} cm

P1.10 (a) and (f); (b) and (d); (c) and (e)

P1.12 $\dfrac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$

P1.14 (a) [A] = L/T^3 and [B] = L/T; (b) L/T

P1.16 667 lb/s

P1.18 9.19 nm/s

P1.20 2.57×10^6 m^3

P1.22 (a) $7.14\times10^{-2}\ \dfrac{\text{gal}}{\text{s}}$; (b) $2.70\times10^{-4}\ \dfrac{\text{m}^3}{\text{s}}$; (c) 1.03 h

P1.24 290 m^3, 2.9×10^8 cm^3

P1.26 $r_{\text{Fe}}(1.43)$

P1.28 (a) 3.39×10^5 ft^3; (b) 2.54×10^4 lb

P1.30 (a) 2.07 mm; (b) 8.62×10^{13} times as large

P1.32 (a) ~ 10^2 kg; (b) ~ 10^3 kg

P1.34 10^7 rev

P1.36 (a) 3; (b) 4; (c) 3; (d) 2

P1.38 (a) 796; (b) 1.1; (c) 17.66

P1.40 9 bars / year

P1.42 1.66×10^3 kg/m^3

P1.44 288°; 108°

P1.46 See P1.46 for complete description.

P1.48 1.38×10^3 m

P1.50 (a) nine times smaller; (b) Δt is inversely proportional to the square of d; (c) Plot Δt on the vertical axis and $1/d^2$ on the horizontal axis; (d) $4QL/k\pi\left(T_h - T_c\right)$

P1.52 $1.61\times10^3\ \text{kg/m}^3,\ 0.166\times10^3\ \text{kg/m}^3,\ (1.61\pm0.17)\times10^3\ \text{kg/m}^3$

P1.54 3.64 cents; the cost is negligible compared to $4.98.

P1.56 (a) 10^{14} bacteria; (b) beneficial

P1.58 The scenario has the contestants succeeding on the whole. But the calculation shows that is impossible. It just takes too long!

P1.60 $h = r\tan\phi = (\tan\theta)C/2\pi$

P1.62 10^{11} stars

P1.64 (a) m = 346 g – (14.5 g/cm^3)a^3; (b) a = 0; (c) 346 g; (d) yes; (e) no change

P1.66 (a) 478 cm^3/s; (b) 0.225 cm/s; (c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.68 31.0°

P1.70 (a-b) see ANS. FIG. P1.70(a) and P1.70(b); (c) $y = x$ tan12.0° and $y = (x - 1.00$ km) tan14.0°; (d) $y = 1.44$ km

P1.72 $$\frac{d\tan\phi\tan\theta}{\tan\phi - \tan\theta}$$